Univalent Foundations

Exercises

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Problems on I. Type theory

- Solve the exercises from the lecture (slides on [1]).
- Define addition + of natural numbers in terms of the eliminator, and via pattern matching.
- Give a proof of $\mathsf{Id}(2+2, 4)$. Explain how/why your proof works.
- Given types A, B, and C, define maps between $A \times (B + C)$ and $A \times B + A \times C$. Show that they are pointwise inverses.
- For A, B, and $P : A \to \mathsf{Type}$, give maps between $\sum_{x:A} B \times P(x)$ and $B \times \sum_{x:A} P(x)$. Show that they are pointwise inverses.

If you want to do the exercises in Agda, there is a small file on [1] to get you started.

[1] `http://benedikt-ahrens.de/teaching`

Problems on II. Univalent type theory

- Solve the exercises from the lectures [1].
- Show $\neg(\mathsf{true} \rightsquigarrow \mathsf{false})$. Draw a picture of the proof.
- Construct equivalences
  - $A \times 1 \simeq A$
  - $A \times 0 \simeq 0$
  - $A + 0 \simeq A$
- Show that the identity function $A \to A$ is an equivalence.
  Show that the composition of equivalences is an equivalence.
- Think of some other equivalences that you could construct.
  Construct them.

[1] http://benedikt-ahrens.de/teaching

Problems on III. Set-level mathematics

- Show that the universe Type is not a set.
- Show that the Booleans have decidable path-equality.
- Show that the natural numbers have decidable path-equality.
- Let X, Y, Z be types, let $f : X \to Y$ and $g_1, g_2 : Y \to Z$. Let f be surjective, and let Z be a set. Suppose that, for any $x : X$, you have $g_1(f(x)) \rightsquigarrow g_2(f(x))$. Show that, for any $y : Y$, $g_1(y) \rightsquigarrow g_2(y)$.
- Show that $\sum_{X:\mathsf{Type}} \mathsf{isofhlevel}(n)(X)$ is of level $S(n)$.